Example

$$\lim_{b\to 3} \frac{(\sin(b-3))(\sqrt{b+1}+2)}{(\sqrt{b+1}-2)(\sqrt{b+1}+2)}$$

$(A-B)(A+B)$
$=A^2-B^2$

$$= \lim_{b\to 3} \frac{(\sin(b-3))(\sqrt{b+1}+2)}{(b+1)-4}$$

$$= \lim_{b\to 3} \frac{\sin(b-3)}{(b-3)} (\underbrace{\sqrt{3+1}+2}_{2})$$

$(b-3)\to 0$

$\to 1$

$$\boxed{\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1}$$

$$= \boxed{4}$$

$$\lim_{r\to -2^-} \frac{|r^2+5r+6|}{r^2+3r+2}$$

$r=-2.1$
$r=-2.005$
$r=-2.00003$

Tips: ① factor
② $|AB| = |A|\,|B|$

③ $\dfrac{|A|}{A}$ $A \to 0^+$ $\to 1$

$$\lim_{A \to 0^-} \frac{|A|}{A} = -1$$

More examples of limits

① $\displaystyle\lim_{x \to \frac{\pi}{2}} \frac{\sin(x)}{x} = \frac{\sin(\frac{\pi}{2})}{\pi/2} = \frac{1}{\pi/2} = \boxed{\frac{2}{\pi}}.$

② $\displaystyle\lim_{n \to \infty} (-1)^n = DNE)$

$n=1 \to -1$
$n=2 \to 1$
$n=3 \to -1$
$n=4 \to 1$
$\vdots$

No limit.

The numbers 1 & −1 are cluster points (limit points) of the sequence

$\rightarrow$ But no limit.

(3) $$\lim_{\theta \to 0} \frac{\sin(\theta^2)}{1-\cos(3\theta)}$$

plug $\frac{0}{1-1} = \frac{0}{0}$ form.

$$= \lim_{\theta \to 0} \frac{\sin(\theta^2)}{(1-\cos(3\theta))} \frac{(1+\cos(3\theta))}{(1+\cos(3\theta))}$$

$$= \lim_{\theta \to 0} \frac{\sin(\theta^2)(1+\cos(3\theta))}{1-\cos^2(3\theta)}$$

$$= \lim_{\theta \to 0} \frac{\sin(\theta^2)(1+\cos(3\theta))}{\sin^2(3\theta)}$$

$$\rightarrow = \lim_{\theta \to 0} \underbrace{\sin(\theta^2)}_{?} \cdot \frac{1}{\sin^2(3\theta)} \cdot \underbrace{(1+\cos(3\theta))}_{?}$$

$$= \lim_{\theta \to 0} \left(\frac{\theta^2}{(3\theta)^2}\right) \frac{\sin(\theta^2)}{\theta^2} \cdot \frac{1(3\theta)^2}{\sin^2(3\theta)} \cdot (1+\cos(3\theta))$$

$$= \lim_{\theta \to 0} \frac{\theta^2}{9\theta^2} \frac{\sin(\theta^2)}{\theta^2} \cdot \left(\frac{3\theta}{\sin(3\theta)}\right)^2 (1+\cos(3\theta))$$

$$= \lim_{\substack{\theta \to 0 \\ 3\theta \to 0}} \frac{1}{9} \cdot \frac{\sin(\theta^2)}{\theta^2} \cdot \left(\frac{3\theta}{\sin(3\theta)}\right)^2 \cdot (1+\underbrace{\cos(3\theta)}_{1})$$

$$= \frac{1}{9} \cdot 1 \cdot 1 \cdot (1+1) = \boxed{\frac{2}{9}}$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

$$\to \frac{1}{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}} = \frac{1}{1}$$

$$\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1$$

(4) $$\lim_{y\to 0} \frac{49-7^{3y+2}}{y^2+2y}$$

plug $\frac{49-7^2}{0+0}$ $\frac{0}{0}$ form

$$= \lim_{y\to 0} \frac{49-7^{3y}7^{2}}{y(y+2)}$$

49

$49-49\cdot 7^{3y}$

$$= \lim_{y\to 0} \frac{49\left(1-7^{3y}\right)}{y(y+2)}$$

$\boxed{\left(7=e^{\ln 7}\right)}$

$$= \lim_{y\to 0} \frac{49\left(1-\left(e^{\ln 7}\right)^{3y}\right)}{y(y+2)}$$

$$= \lim_{y\to 0} \frac{49\left(1-e^{(\ln 7)3\cdot y}\right)}{y(y+2)}$$

Special limit $\displaystyle\lim_{B\to 0} \frac{e^B - 1}{B} = 1$

$$= \lim_{y\to 0} \frac{-49\left(e^{(\ln 7)3y} - 1\right)}{(\ln 7)3y\,(y+2)} \cdot (\ln 7)3$$

$\to 1$

$$= \lim_{y\to 0} \frac{-49 \cdot 1 \cdot (\ln 7)3}{y+2}$$

$y \to 0$

$$= \frac{-49 \cdot 3 \cdot \ln 7}{2}$$

$$= \boxed{\frac{-147 \ln 7}{2}}$$

⑤ $\lim_{p\to\infty}\left(\frac{p}{p+2}\right)^{3p-2}$

plug $(1)^{\infty}$ Aaah.

undefined.

Think:
looks like
$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$

$\left(\frac{n+1}{n}\right)^{n}$

$= \lim_{p\to\infty}\left(\frac{1}{\left(\frac{p+2}{p}\right)}\right)^{3p-2}$

$\frac{1}{\frac{p+2}{p}} = 1\cdot\frac{p}{p+2}$
$= \frac{p}{p+2}$

$= \lim_{p\to\infty}\left(\frac{1}{\left(1+\frac{2}{p}\right)}\right)^{3p-2}$

$$= \lim_{p \to \infty} \frac{1^{3p-2}}{\left(1+\frac{2}{p}\right)^{3p-2}}$$

$$= \lim_{p \to \infty} \frac{1}{\left(1+\frac{2}{p}\right)^{3p-2}}$$

$$\lim_{n \to \infty} \left(1+\frac{1}{n}\right)^{n} = e$$

$$n = \frac{p}{2}$$

$$= \lim_{p \to \infty} \frac{1}{\left(1+\frac{1}{p/2}\right)^{3p-2}}$$

$$= \lim_{p \to \infty} \frac{1}{\left(\underbrace{\left(1+\frac{1}{p/2}\right)^{(p/2)}}_{e}\right)^{(3p-2)\left(\frac{2}{p}\right)}}$$

$$= \lim_{p \to \infty} \frac{1}{e^{(3p-2)\left(\frac{2}{p}\right)}}$$

$$\text{exponent} = (3p-2)\left(\frac{2}{p}\right) = \frac{2\cdot(3p-2)}{p}$$

$$= \frac{2\cdot(3p-2)\left(\frac{1}{p}\right)}{p\left(\frac{1}{p}\right)}$$

$$= \frac{2\left(3 - \underbrace{\frac{2}{p}}_{\to 0}\right)}{1} \Rightarrow 6$$

$$= \lim_{p \to \infty} \frac{1}{e^{6}} = \boxed{\frac{1}{e^{6}}}$$